

40S

Chemistry

Solubility

1

Outcomes . . .

- ▶ Describe and write a balanced chemical equation to represent the equilibrium in a saturated aqueous solution of an ionic compound.
- ▶ Write a solubility product expression, given a balanced chemical equation for a solubility reaction.
- ▶ Distinguish between solubility and solubility product constant (K_{sp}).
- ▶ Calculate the solubility product, given the solubility of a compound in water, and vice versa.

2

Introduction

- ▶ When placed into water, slightly soluble substances establish an equilibrium between the solid and dissolved ions in a saturated solution. This equilibrium is described by the solubility product.
- ▶ Using the solubility product, we can calculate the ion concentrations and the solubility of the substance.

3

- ▶ when a solution is saturated, there exists an equilibrium between the dissolved solute particles and the solid solute particles.
- ▶ For an ionic compound, such as sodium chloride, we express the equilibrium in terms of a chemical equation:
 - ▶ $\text{NaCl}(s) \rightleftharpoons \text{Na}^+(aq) + \text{Cl}^-(aq)$
- ▶ This equilibrium is dynamic, since the rate of dissolving of each ion is equal to the crystallization of each ion.

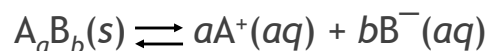
4

The Solubility Product

- ▶ Substances chemists believed to be insoluble in water, by experiment, were shown to be slightly soluble.
- ▶ With sensitive instruments, chemists are able to show that substances that would not dissolve in water actually do dissolve to a very small extent.

5

- ▶ When a sparingly soluble ionic solid is dissolved in water to form a saturated solution the general equilibrium equation is



- ▶ Where A is a positively charged ion and B is a negatively charged ion.

6

- ▶ At a given temperature, the equilibrium law for this reaction is given as

$$K_C = \frac{[A^+]^a [B^-]^b}{[A_a B_b]}$$

$$K_C \cdot [A_a B_b] = [A^+]^a [B^-]^b$$

- ▶ However, the term $K_C[A_a B_b]$ can be replaced by a new constant, K_{sp} , called the **solubility product**.

7

$$K_{sp} = [A^+]^a [B^-]^b$$

- ▶ The **solubility product constant** is the product of ion concentrations in a saturated solution. The solubility product constant takes into account the presence of the solid.

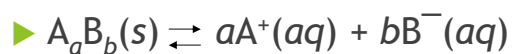
8

The Solubility Product Expression

- ▶ Recall that the general form of the solubility product expression is

$$K_{sp} = [A^+]^a[B^-]^b$$

- ▶ for the dissociation equation



9

Example 1

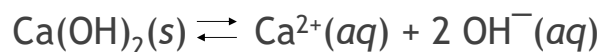
- ▶ Write the dissociation equation and the expression for the solubility product constant for calcium hydroxide.

Solution

- ▶ Write the chemical formula for calcium hydroxide, $\text{Ca}(\text{OH})_2$.

10

- ▶ Write the dissociation equation, remembering that the subscript indicates two hydroxide ions dissociate for every calcium ion:



- ▶ When writing the solubility product expression, the molar coefficient in the dissociation equation becomes the exponent.

- ▶ $K_{sp} = [\text{Ca}^{2+}][\text{OH}^{-}]^2$

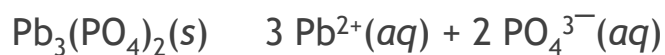
11

Example 2

- ▶ Write the solubility product expression for $\text{Pb}_3(\text{PO}_4)_2$.

- ▶ **Solution**

- ▶ Write the dissociation equation first.



- ▶ Use the molar coefficients to write the solubility product expression.

$$K_{sp} = [\text{Pb}^{2+}]^3[\text{PO}_4^{3-}]^2$$

12

Calculating Solubility Product

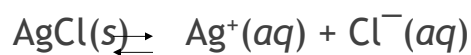
Example 3

- ▶ If at equilibrium, the concentration of silver ions is 1.3×10^{-5} mol/L and the concentration of chloride ions is 1.3×10^{-5} mol/L, what is the K_{sp} of silver chloride?

13

▶ Solution

- ▶ Write out the dissociation equation:



- ▶ Write the solubility product expression:

$$K_{sp} = [\text{Ag}^+][\text{Cl}^-]$$

- ▶ Substitute given values:

$$\text{▶ } K_{sp} = (1.3 \times 10^{-5})(1.3 \times 10^{-5})$$

$$\text{▶ } K_{sp} = 1.7 \times 10^{-10}$$

***NOTE: K_{sp} has no units.**

14

Solubility

- ▶ Solubility and solubility product are two different terms.
- ▶ **Solubility** is the maximum amount of solute that can dissolve in a certain amount of solvent at a certain temperature.
- ▶ Solubility has an infinite number of possible values, depending on temperature and other solutes present.

15

- ▶ **Solubility product** is an equilibrium constant and has only one value for a given solid at a particular temperature.
- ▶ **Example 4**
- ▶ The solubility of PbF_2 is 0.466 g/L. What is the value of the solubility product?

16

Solution

Step 1: Determine the number of moles/L of PbF_2 .

$$\text{PbF}_2 = 207.19\text{g} + 2(19.00\text{g}) = 245.19\text{g}$$

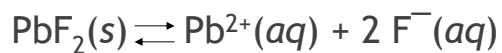
$$0.466\text{g/L PbF}_2 \times 1 \text{ mol PbF}_2 =$$

$$\frac{1.90 \times 10^{-3} \text{ mol/L}}{245.19\text{g PbF}_2}$$

So, the concentration of PbF_2 is $1.90 \times 10^{-3} \text{ mol/L}$.

17

► **Step 2: Write the dissociation equation.**



18

- ▶ **Step 3: Calculate ion concentrations.**
- ▶ The concentration of PbF_2 and Pb^{2+} ions will be equal since their stoichiometry is 1:1.
- ▶ $[\text{PbF}_2] = [\text{Pb}^{2+}] = 1.90 \times 10^{-3} \text{ mol/L}$
- ▶ From the stoichiometry,
- ▶ $[\text{F}^-] = 2 \times [\text{Pb}^{2+}] = 3.80 \times 10^{-3} \text{ mol/L}$

19

- ▶ **Step 4: Write the solubility product expression and calculate.**
- ▶ $K_{\text{sp}} = [\text{Pb}^{2+}][\text{F}^-]^2$
- ▶ $K_{\text{sp}} = (1.90 \times 10^{-3})(3.80 \times 10^{-3})^2$
- ▶ **$K_{\text{sp}} = 2.74 \times 10^{-8}$**
- ▶ Therefore, the solubility product of PbF_2 is 2.74×10^{-8} .

20

Determining Ion Concentrations from K_{sp}

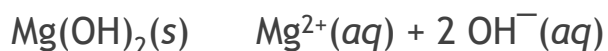
Example 5

- ▶ The K_{sp} of magnesium hydroxide is 8.9×10^{-12} . What will be the equilibrium concentrations of the dissolved ions in a saturated solution of $Mg(OH)_2$?

21

Solution

Step 1: Write the dissociation equation.



Step 2: Assign variables to the ion [].

- ▶ The initial concentrations of the ions, before dissolving, is zero.
- ▶ The equilibrium concentrations are unknown, so we will assign them a value of "x".

22

- ▶ The OH^- is assigned a value of $2x$ because of the stoichiometry.

Step 3: Substitute values into K_{sp} expression and solve for x .

$$K_{\text{sp}} = [\text{Mg}^{2+}] [\text{OH}^-]^2$$

$$8.9 \times 10^{-12} = (x) (2x)^2$$

$$8.9 \times 10^{-12} = 4x^3$$

$$\frac{8.9 \times 10^{-12}}{4} = \frac{4x^3}{4}$$

$$1.3 \times 10^{-4} = x$$

23

Step 4: Calculate ion concentrations from the value of x .

- ▶ $[\text{Mg}^{2+}] = x = 1.3 \times 10^{-4} \text{ mol/L}$
- ▶ $[\text{OH}^-] = 2x = 2.6 \times 10^{-4} \text{ mol/L}$

24

In Summary . . .

- ▶ Substances which are insoluble are actually slightly soluble.
- ▶ The solubility product, K_{sp} , describes the product of ion concentrations in saturated solutions.
- ▶ Solubility can be determined from the solubility product.

25

Assignment

- ▶ Complete Solubility *Assignment 1*
- ▶ Complete Solubility *Assignment 2*

26

40S

Chemistry

Solubility Rules

27

Solubility Rules

- ▶ Insoluble compounds are generally described as those that precipitate upon mixing equal volumes of solutions which are 0.10 mol/L in the respective ions.
- ▶ A **precipitate** is the solid that forms when two solutions are mixed. A precipitate will usually cause a mixture to appear cloudy.
- ▶ [Video 1](#)
- ▶ [Video 2](#)

28

Predicting if a Precipitate Forms

- ▶ Not all reactions produce a precipitate. If the volume of solution is large enough, and the amount of solute is small enough no precipitate will form.
- ▶ The **reaction quotient**, Q_{sp} , to predict whether a precipitate forms. The form of the reaction quotient is the same as the K_{sp} .

29

- ▶ By comparing the value of the reaction quotient with the solubility product, we can determine if an aqueous solution is saturated or unsaturated . . .

30

If,

- ▶ $Q_{sp} = K_{sp}$, the solution is just saturated and no precipitate forms.
- ▶ $Q_{sp} > K_{sp}$, the solution is saturated and a **precipitate forms**.
- ▶ $Q_{sp} < K_{sp}$, the solution is unsaturated and no precipitate forms.

31

Determining if a Solution is Saturated

- ▶ The reaction quotient can be used to determine if adding a specific amount of solute to a solvent produces a saturated solution.

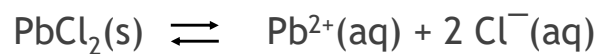
Example 4

- ▶ The K_{sp} of lead (II) chloride is 1.6×10^{-5} . If 0.57 g of lead (II) chloride are added to 1500 mL of water, is the solution saturated? Assume no volume change.

32

Solution

Step 1: Write out the dissociation equation for lead (II) chloride.



33

Step 2: Determine the concentration of the PbCl_2 .

$$\text{PbCl}_2 = 207.19\text{g/mol} + 2(35.45\text{g/mol}) = 278.09\text{g/mol}$$

$$0.57\text{g PbCl}_2 \times \frac{1 \text{ mol PbCl}_2}{278.09\text{g PbCl}_2} = 0.00204 \text{ mol PbCl}_2$$

$$\frac{0.57\text{g PbCl}_2}{278.09\text{g PbCl}_2}$$

$$M = \frac{\text{mol}}{\text{L}} = \frac{0.00204 \text{ mol PbCl}_2}{1.5\text{L}} = 1.37 \times 10^{-3} \text{ mol/L}$$

$$\frac{0.00204 \text{ mol PbCl}_2}{1.5\text{L}}$$

34

Step 3: Calculate the value of the reaction quotient.

Determine the concentration of each ion:

$$[\text{Pb}^{2+}] = [\text{PbCl}_2] = 1.37 \times 10^{-3} \text{ mol/L}$$

$$[\text{Cl}^-] = 2 \times [\text{Pb}^{2+}] = 2.74 \times 10^{-3} \text{ mol/L}$$

$$Q_{\text{sp}} = [\text{Pb}^{2+}] [\text{Cl}^-]^2$$

$$Q_{\text{sp}} = (1.37 \times 10^{-3}) (2.74 \times 10^{-3})^2$$

$$Q_{\text{sp}} = 1.03 \times 10^{-8} = 1.0 \times 10^{-8}$$

Since $Q_{\text{sp}} < K_{\text{sp}}$, the solution is unsaturated.

35

Does a Precipitate Form?

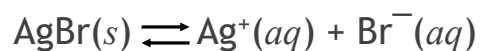
Example 5

If 20.0 mL of a 0.0010 mol/L silver nitrate solution is mixed with 20.0 mL of a 3.0×10^{-5} mol/L potassium bromide solution, does silver bromide ($K_{\text{sp}} = 5.0 \times 10^{-13}$) precipitate? Assume the volumes are additive.

36

Solution

Step 1: Write the dissociation equation for silver bromide.



37

Step 2: Determine the ion concentrations.

Since the solutions are mixed, we will assume the new volume will be the sum of the two.

New volume = 20.0 mL + 20.0 mL = 40.0 mL or 0.0400 L

38

Calculate the silver ion and bromide ion concentrations by using the dilution formula:

$$[\text{Ag}^+] = \frac{C_1 V_1}{V_2} = \frac{(0.0010 \text{ mol/L})(0.0200 \text{ L})}{(0.0400 \text{ L})} = 5.0 \times 10^{-4} \text{ mol/L Ag}^+$$

$$[\text{Br}^-] = \frac{C_1 V_1}{V_2} = \frac{(3.0 \times 10^{-5} \text{ mol/L})(0.0200 \text{ L})}{(0.0400 \text{ L})} = 1.5 \times 10^{-5} \text{ mol/L Br}^-$$

39

Step 3: Calculate the reaction quotient.

$$\begin{aligned} Q_{\text{sp}} &= [\text{Ag}^+][\text{Br}^-] \\ &= (5.0 \times 10^{-4})(1.5 \times 10^{-5}) \\ &= 7.5 \times 10^{-9} \end{aligned}$$

$Q_{\text{sp}} > K_{\text{sp}}$, the solution is saturated and a precipitate forms.

40

In Summary . . .

- ▶ Substances which are insoluble are actually slightly soluble.
- ▶ The solubility product, K_{sp} , describes the product of ion concentrations in saturated solutions.
- ▶ Solubility can be determined from the solubility product.

41

Lesson Summary

- ▶ When two solutions of ionic compounds are mixed, a solid with low solubility may precipitate from the mixture.
- ▶ The trial K_{sp} , or reaction quotient can be used to predict if a precipitate forms.

42

Assignment

- ▶ Complete Solubility *Assignment 3*

43

40S Chemistry

Common Ion Effect

44

Introduction

- ▶ we have studied the solubility of ionic solids in pure water and precipitates from mixtures.
- ▶ What happens to solubility of an ionic compound if the water contains an ion in common with the ionic solid?

45

Outcome

- ▶ Calculate the solubility of an ionic compound in the presence of an ion in common with the ionic compound.

46

Common Ions

- ▶ When an ionic compound dissolves in pure water, the initial concentration of each ion is zero.
- ▶ If an ionic compound dissolves in a solution that has an ion in common with the compound, this is not the case.
- ▶ Even though the starting concentrations may not be zero, the product of the ions must still equal the solubility product constant.

47

- ▶ How would the solubility of silver chloride in pure water change if we try dissolving it in tap water?



- ▶ Tap water often has chlorine added to kill bacteria. The chlorine exists as chloride ions, so when we dissolve silver chloride in tap water, chloride ions are present.

48

- ▶ According to Le Chatelier's Principle, Adding more chloride ions to a saturated solution would cause the equilibrium to shift to the left to use up the excess product.
- ▶ This would result in more solid formed, and a decreased solubility.

49

Solubility in the Presence of a Common Ion

- ▶ Le Chatelier's Principle predicts that the solubility of an ionic solid in a solution containing a common ion decreases its solubility.
- ▶ Will our calculations support this?

50

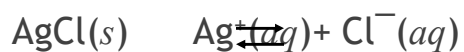
Example 1

- Determine the solubility of silver chloride in pure water and in a solution of 0.10 mol/L sodium chloride. The K_{sp} of AgCl is 1.7×10^{-10} .

51

Solution

Step 1: Solubility of AgCl in pure water.



$$K_{sp} = [\text{Ag}^+][\text{Cl}^-] = 1.7 \times 10^{-10}$$

$$[\text{Ag}^+] = [\text{Cl}^-] = x$$

x = molar solubility

$$(x)(x) = 1.7 \times 10^{-10}$$

$$x^2 = 1.7 \times 10^{-10}$$

$$x = 1.3 \times 10^{-5}$$

Since $[\text{AgCl}] = x$, the solubility of AgCl in pure water is 1.3×10^{-5} mol/L.

52

Step 2: Determine the $[\text{Cl}^-]$ in solution.

- ▶ The sodium chloride solution will have an initial chloride ion concentration of 0.10 mol/L.
- ▶ This must be added to the $[\text{Cl}^-]$ from the AgCl solution . . .

53

$$K_{sp} = [\text{Ag}^+] [\text{Cl}^-] = 1.7 \times 10^{-10}$$

Let x = molar solubility

$$[\text{Ag}^+] = x, [\text{Cl}^-] = 0.10 + x$$

$$(x) (0.10 + x) = 1.7 \times 10^{-10}$$

$$0.10x = 1.7 \times 10^{-10}$$

$$x = 1.7 \times 10^{-9} \text{ mol/L}$$

The value of x is considered insignificant because the K_{sp} is so small

54

- ▶ AgCl is 1.7×10^{-9} mol/L in 0.10 mol/L NaCl and 1.3×10^{-5} mol/L in pure water.
- ▶ The solubility decreases, just as predicted.

55

Solubility in the Presence of a Common Ion

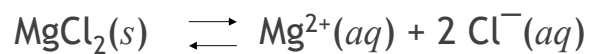
Example 2

- ▶ The K_{sp} of lead (II) chloride, $PbCl_2$, is 1.6×10^{-5} . What is the solubility of lead (II) chloride in a 0.10 mol/L solution of magnesium chloride, $MgCl_2$?

56

Solution

Step 1: Determine the concentration of the common ion, Cl^- .



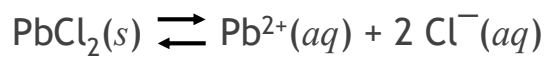
- ▶ From the dissociation of magnesium chloride,
- ▶ $[\text{Cl}^-] = 2 \times [\text{MgCl}_2] = 0.20 \text{ mol/L}$

57

Step 2: Determine the $[\text{Cl}^-]$ in solution.

- ▶ The magnesium chloride solution will have an initial chloride ion concentration of 0.20 mol/L.
- ▶ This must be added to the $[\text{Cl}^-]$ from the PbCl_2 solution . . .

58



$$K_{sp} = [\text{Pb}^{2+}] [\text{Cl}^{-}]^2 = 1.6 \times 10^{-5}$$

Let x = molar solubility

$$[\text{Pb}^{2+}] = x, [\text{Cl}^{-}] = 0.20 + 2x$$

$$(x) (0.20 + 2x)^2 = 1.6 \times 10^{-5}$$

$$0.040x = 1.6 \times 10^{-5}$$

$$x = 4.0 \times 10^{-4} \text{ mol/L}$$

The value of x is considered insignificant because the K_{sp} is so small

59

- The solubility of PbCl_2 is $4.0 \times 10^{-4} \text{ mol/L}$ in $0.10 \text{ mol/L MgCl}_2$ solution.

60

Solubility in the Presence of a Common Ion

Example 3

- ▶ The K_{sp} of lead (II) chloride is 1.6×10^{-5} . What is the solubility of lead (II) chloride in a 0.10 mol/L solution of lead (II) nitrate, $Pb(NO_3)_2$?

61

Solution

Step 1: Determine the concentration of the common ion, Pb^{2+}

- ▶ $Pb(NO_3)_2(s) \rightleftharpoons Pb^{2+}(aq) + 2 NO_3^-(aq)$
- ▶ From the dissociation of lead (II) nitrate,
- ▶ $[Pb^{2+}] = [Pb(NO_3)_2] = 0.10 \text{ mol/L}$

62

Step 2: Determine the $[\text{Pb}^{2+}]$ in solution.

- ▶ The lead (II) chloride solution will have an initial lead ion concentration of 0.10 mol/L.
- ▶ This must be added to the $[\text{Pb}^{2+}]$ from the $\text{Pb}(\text{NO}_3)_2$ solution . . .

63

$$K_{sp} = [\text{Pb}^{2+}] [\text{Cl}^-]^2 = 1.6 \times 10^{-5}$$

Let x = molar solubility

$$[\text{Pb}^{2+}] = 0.10 + x, [\text{Cl}^-] = 2x$$

$$(2x)^2 (0.10 + x) = 1.6 \times 10^{-5}$$

$$0.40x^2 = 1.6 \times 10^{-5}$$

$$x = 6.3 \times 10^{-3} \text{ mol/L}$$

The value of x is considered insignificant because the K_{sp} is so small

64

- ▶ The solubility of PbCl_2 is 6.3×10^{-3} mol/L in 0.10 mol/L $\text{Pb}(\text{NO}_3)_2$.

65

Alternate Method:

- ▶ K_{sp} problems can also be solved using an ICE table.
- ▶ The solution, with ICE table, for **Example 3** is shown on the following slides.
- ▶ The first step and some of step 2 are identical to the original solution. But as we progress further . . .

66

Solution

Step 1: Determine the concentration of the common ion, Pb^{2+}

- ▶ $\text{Pb}(\text{NO}_3)_2(s) \rightleftharpoons \text{Pb}^{2+}(aq) + 2 \text{NO}_3^-(aq)$
- ▶ From the dissociation of lead (II) nitrate,
- ▶ $[\text{Pb}^{2+}] = [\text{Pb}(\text{NO}_3)_2] = 0.10 \text{ mol/L}$

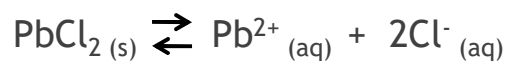
67

Step 2: Determine the $[\text{Pb}^{2+}]$ in solution.

- ▶ The lead (II) chloride solution will have an initial chloride ion concentration of 0.10 mol/L.
- ▶ This must be added to the $[\text{Pb}^{2+}]$ from the $\text{Pb}(\text{NO}_3)_2$ solution . . .

68

► Set up an ICE table:



I	Solid	0.10	0
C	Solid	+x	+2x
E	solid	0.10 + x	2x

69

I	Solid	0.10	0
C	Solid	+x	+2x
E	solid	0.10 + x	2x

$$K_{sp} = [\text{Pb}^{2+}] [\text{Cl}^-]^2 = 1.6 \times 10^{-5}$$

$$(0.10 + x) (2x)^2 = 1.6 \times 10^{-5}$$

$$0.40x^2 \approx 1.6 \times 10^{-5}$$

$$x = 6.3 \times 10^{-3} \text{ mol/L}$$

The value of x is considered insignificant because the K_{sp} is so small

The solubility of PbCl_2 is $6.3 \times 10^{-3} \text{ mol/L}$ in $0.10 \text{ mol/L Pb(NO}_3)_2$.

70

Lesson Summary

- ▶ The solubility of an ionic solid decreases in the presence of an ion in common with the ionic solid.
- ▶ Solving problems requires determining the molar solubility and substituting values into the solubility product expression.

71

Assignment

- ▶ Complete worksheets: *K_{sp} Assignments #1-5*

72